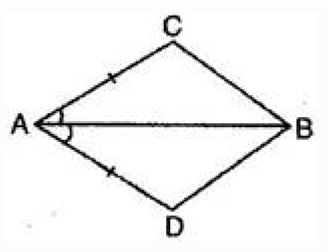
Exercise 7.1 (Revised) - Chapter 7 - Triangles - Ncert Solutions class 9 - Maths

Updated On 11-02-2025 By Lithanya

Chapter 7 - Triangles | NCERT Solutions for Class 9 Maths

Ex 7.1 Question 1.

In quadrilateral $ABCD$ (See figure). $AC = AD$ and AB bisects $\angle A$. Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD?



Answer.

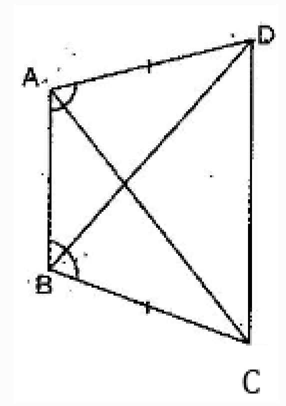
Given: In quadrilateral $\mathrm{ABCD}, \mathrm{AC} = \mathrm{AD}$ and AB bisects $\angle \mathrm{A}$.

To prove: $\triangle \mathrm{ABC} \cong \triangle \mathrm{ABD}$

Proof: In $\triangle \mathrm{ABC}$ and $\triangle \mathrm{ABD}$,
$\mathrm{AC} = \mathrm{AD}$[Given]
$\angle \mathrm{BAC} = \angle \mathrm{BAD}$[$\because$ AB bisects $\angle \mathrm{A}$]
$\mathrm{AB} = \mathrm{AB}$[Common]
$\therefore \triangle \mathrm{ABC} \cong \triangle \mathrm{ABD}$ [By SAS congruency]

Thus $\mathrm{BC} = \mathrm{BD}$ [By C.P.C.T.]
Ex 7.1 Question 2.

ABCD is a quadrilateral in which $\mathrm{AD} = \mathrm{BC}$ and $\angle \mathrm{DAB} = \angle \mathrm{CBA}$. (See figure). Prove that:



(i) $\triangle$ ABD $\cong \triangle BAC$
(ii) $BD = AC$
(iii) $\angle \mathrm{ABD} = \angle \mathrm{BAC}$

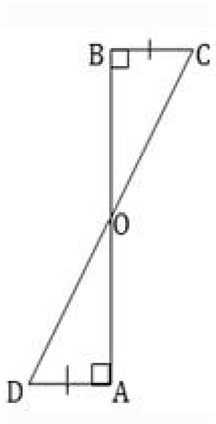
Answer.

(i) In $\triangle \mathrm{ABC}$ and $\triangle \mathrm{ABD}$,
$\mathrm{BC} = \mathrm{AD}$[Given]
$\angle \mathrm{DAB} = \angle \mathrm{CBA}$[Given]
$\mathrm{AB} = \mathrm{AB}$[Common]
$\therefore \triangle \mathrm{ABC} \cong \triangle \mathrm{ABD}$ [By SAS congruency]

Thus AC = BD [By C.P.C.T.]
(ii) Since $\triangle ABC \cong \triangle ABD$
$\therefore AC = BD[By$ C.P.C.T.]
(iii) Since $\triangle \mathrm{ABC} \cong \triangle \mathrm{ABD}$
$\therefore \angle \mathrm{ABD} = \angle \mathrm{BAC}$ [By C.P.C.T.]

Ex 7.1 Question 3.

AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB (See figure)

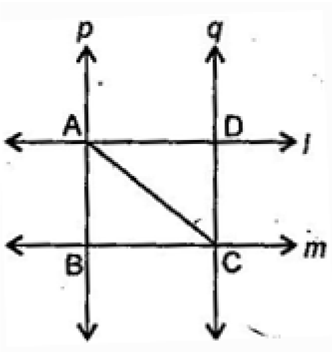


Answer.

In $\triangle BOC$ and $\triangle AOD$,
$\angle \mathrm{OBC} = \angle \mathrm{OAD} = 90^{\circ}$ [Given]
$\angle \mathrm{BOC} = \angle \mathrm{AOD}$ [Vertically Opposite angles]
$\mathrm{BC} = \mathrm{AD}[$ Given $]$
$\therefore \triangle \mathrm{BOC} \cong \triangle \mathrm{AOD}$ [By ASA congruency]
$\Rightarrow \mathrm{OB} = \mathrm{OA}$ and $\mathrm{OC} = \mathrm{OD}[$ By C.P.C.T. $]$

Ex 7.1 Question 4.

l and m are two parallel lines intersected by another pair of parallel lines p and q (See figure). Show that $\triangle ABC \cong \triangle CDA$.



Answer.

AC being a transversal. [Given]

Therefore $\angle \mathrm{DAC} = \angle \mathrm{ACB}$ [Alternate angles]

Now $p \| q$ [Given]

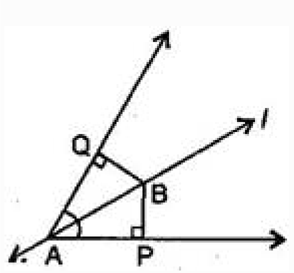
And AC being a transversal. [Given]

Therefore $\angle \mathrm{BAC} = \angle \mathrm{ACD}$ [Alternate angles]

Now in $\triangle \mathrm{ABC}$ and $\triangle \mathrm{ADC}$,
$\angle \mathrm{ACB} = \angle \mathrm{DAC}$ [Proved above]
$\angle \mathrm{BAC} = \angle \mathrm{ACD}$ [Proved above]
$\mathrm{AC} = \mathrm{AC}[$ Common $]$
$\therefore \triangle \mathrm{ABC} \cong \triangle \mathrm{CDA}$ [By ASA congruency]

Ex 7.1 Question 5.

Line l is the bisector of the angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle \mathbf{A}$. Show that:



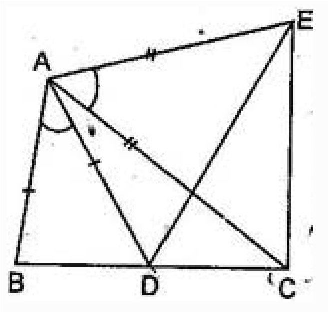
(i) $\triangle \mathrm{APB} \cong \triangle AQB$
(ii) $\mathrm{BP} = \mathrm{BQ}$ or P is equidistant from the arms of $\angle \mathrm{A}$ (See figure).

Answer.

Given: Line l bisects $\angle A$.
$\therefore \angle BAP = \angle BAQ$
(i) In $\triangle ABP$ and $\triangle ABQ$,
$\angle BAP = \angle BAQ$ [Given]
$\angle BPA = \angle BQA = 90^\circ$ [Given]
$AB = AB$ [Common]
$\therefore \triangle APB \cong \triangle AQB$ [By ASA congruency]
(ii) Since $\triangle APB \cong \triangle AQB$
$\therefore BP = BQ$[By C.P.C.T.]
$\Rightarrow$ B is equidistant from the arms of $\angle A$.

Ex 7.1 Question 6.

In figure, $AC = AB$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$. $\wedge$



Answer.

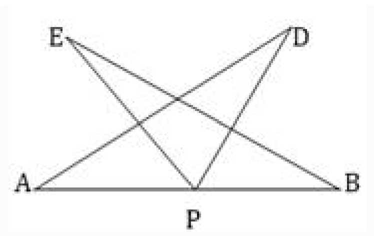
Given that $\angle BAD = \angle EAC$

Adding $\angle$ DAC on both sides, we get
$\angle BAD + \angle DAC = \angle EAC + \angle DAC$
$\Rightarrow \angle BAC = \angle EAD \ldots\ldots\ldots$ (i)

Now in $\triangle ABC$ and $\triangle AED$,
$AB = AD$[Given]
$AC = AE$[Given]
$\angle BAC = \angle DAE$ [From eq. (i)]
$\therefore \triangle ABC \cong \triangle ADE$ [By SAS congruency]
$\Rightarrow BC = DE$[By C.P.C.T.]

Ex 7.1 Question 7.

AB is a line segment and P is the mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that:
(i) $\triangle$ DAF $\cong \triangle FBP$
(ii) $AD = BE$ (See figure)



Answer.

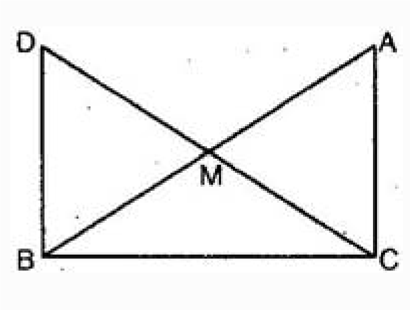
Given that $\angle EPA = \angle DPB$

Adding $\angle$ EPD on both sides, we get
$\angle EPA + \angle EPD = \angle DPB + \angle EPD$
$\Rightarrow \angle APD = \angle BPE \ldots\ldots\ldots$ (i)

Now in $\triangle APD$ and $\triangle BPE$,
$\angle PAD = \angle PBE [\because \angle BAD = \angle ABE$ (given),
$\therefore \angle PAD = \angle PBE]$
$AP = PB$[P is the mid-point of AB]
$\angle APD = \angle BPE$ [From eq. (i)]
$\therefore \triangle DPA \cong \triangle EBP$[By ASA congruency]
$\Rightarrow AD = BE$ [By C.P.C.T.]

Ex 7.1 Question 8.

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B. (See figure)



Show that:
(i) △ AMC ≅ △ BMD
(ii) $\angle \mathrm{DBC}$ is a right angle.
(iii) $\triangle \mathrm{DBC} \cong \triangle \mathrm{ACB}$
(iv) $CM = \frac{1}{2}\mathrm{AB}$

Answer.

(i) In $\triangle \mathrm{AMC}$ and $\triangle \mathrm{BMD}$,
$AM = BM[AB$ is the mid-point of $AB]$
$\angle \mathrm{AMC} = \angle \mathrm{BMD}$ [Vertically opposite angles]
$\mathrm{CM} = \mathrm{DM}[$ Given]
$\therefore \triangle \mathrm{AMC} \cong \triangle \mathrm{BMD}$ [By SAS congruency]
$\therefore \angle \mathrm{ACM} = \angle \mathrm{BDM}$
$\angle \mathrm{CAM} = \angle \mathrm{DBM}$ and $\mathrm{AC} = \mathrm{BD}$ [By C.P.C.T.]
(ii) For two lines AC and DB and transversal DC, we have,
$\angle \mathrm{ACD} = \angle \mathrm{BDC}$ [Alternate angles]

$\therefore AC \| DB$

Now for parallel lines AC and DB and for transversal BC.
$\angle \mathrm{DBC} = \angle \mathrm{ACB}$ [Alternate angles]

But $\triangle \mathrm{ABC}$ is a right angled triangle, right angled at C.
$\therefore \angle \mathrm{ACB} = 90^{\circ}$

Therefore $\angle \mathrm{DBC} = 90^{\circ}$ [Using eq. (ii) and (iii)]
$\Rightarrow \angle \mathrm{DBC}$ is a right angle.
(iii) Now in $\triangle \mathrm{DBC}$ and $\triangle \mathrm{ABC}$,
$DB = AC[$ Proved in part (i)]
$\angle \mathrm{DBC} = \angle \mathrm{ACB} = 90^{\circ}$ [Proved in part (ii)]
$\mathrm{BC} = \mathrm{BC}[$ Common $]$
$\therefore \triangle \mathrm{DBC} \cong \triangle \mathrm{ACB}$ [By SAS congruency]
(iv) Since $\triangle \mathrm{DBC} \cong \triangle \mathrm{ACB}$ [Proved above]
$\therefore \mathrm{DC} = \mathrm{AB}$

$$\Rightarrow AM + CM = AB$$
$$\Rightarrow CM + CM = AB[\because DM = CM]$$
$$\Rightarrow 2CM = AB$$
$$\Rightarrow CM = \frac{1}{2}AB$$

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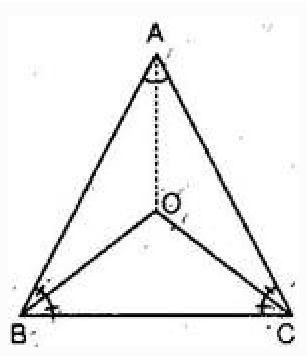
Ex 7.2 Question 1.

In an isosceles triangle $\mathbf{ABC}$, with $\mathbf{AB}=\mathbf{AC}$, the bisectors of $\angle\mathbf{B}$ and $\angle\mathbf{C}$ intersect each other at O. Join A to O. Show that:
(i) $\mathrm{OB}=\mathrm{OC}$
(ii) AO bisects $\angle\mathrm{A}$.

Answer.

(i) ABC is an isosceles triangle in which $AB=AC$.



$\therefore \angle\mathrm{C}=\angle\mathrm{B}$ [Angles opposite to equal sides]
$\Rightarrow \angle\mathrm{OCA}+\angle\mathrm{OCB}=\angle\mathrm{OBA}+\angle\mathrm{OBC}$
$\because$ OB bisects $\angle\mathrm{B}$ and OC bisects $\angle\mathrm{C}$
$\therefore \angle\mathrm{OBA}=\angle\mathrm{OBC}$ and $\angle\mathrm{OCA}=\angle\mathrm{OCB}$
$\Rightarrow \angle\mathrm{OCB}+\angle\mathrm{OCB}=\angle\mathrm{OBC}+\angle\mathrm{OBC}$
$\Rightarrow 2\angle\mathrm{OCB}=2\angle\mathrm{OBC}$
$\Rightarrow \angle\mathrm{OCB}=\angle\mathrm{OBC}$

Now in $\triangle\mathrm{OBC}$,
$\angle\mathrm{OCB}=\angle\mathrm{OBC}$ [Prove above]
$\therefore \mathrm{OB}=\mathrm{OC}$ [Sides opposite to equal sides]
(ii) In $\triangle\mathrm{AOB}$ and $\triangle\mathrm{AOC}$,
$\mathrm{AB}=\mathrm{AC}[$ Given $]$
$\angle\mathrm{OBA}=\angle\mathrm{OCA}$ [Given]

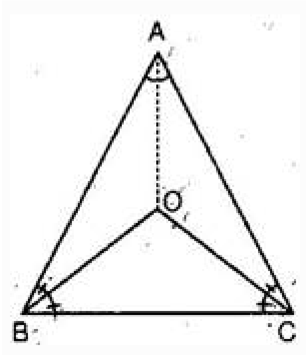
And $\angle\mathrm{B}=\angle\mathrm{C}$

$$\Rightarrow \frac{1}{2}\angle\mathrm{B}=\frac{1}{2}\angle\mathrm{C}$$
$$\Rightarrow \angle\mathrm{OBA}=\angle\mathrm{OCA}$$
$$\Rightarrow \mathrm{OB}=\mathrm{OC}\ [\text{Prove above}]$$
$$\therefore \triangle\mathrm{AOB}\cong\triangle\mathrm{AOC}\ [\text{By SAS congruency}]$$
$$\Rightarrow \angle\mathrm{OAB}=\angle\mathrm{OAC}\ [\text{By C.P.C.T.}]$$

Hence AO bisects $\angle\mathrm{A}$.

Ex 7.2 Question 2.

In $\triangle ABC$, AD is the perpendicular bisector of BC (See figure). Show that $\triangle ABC$ is an isosceles triangle in which $AB=AC$.



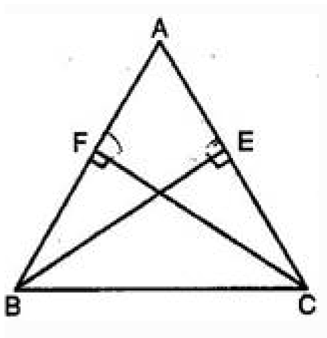
Answer.

In $\triangle AOB$ and $\triangle AOC$,
$BD = CD [AD$ bisects $BC]$
$\angle ADB = \angle ADC = 90^{\circ} [AD \perp BC]$
$AD = AD [Common]$
$\therefore \triangle ABD \cong \triangle ACD$ [By SAS congruency]
$\Rightarrow AB = AC$ [By C.P.C.T.]

Therefore, ABC is an isosceles triangle.

Ex 7.2 Question 3.

ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively (See figure). Show that these altitudes are equal.

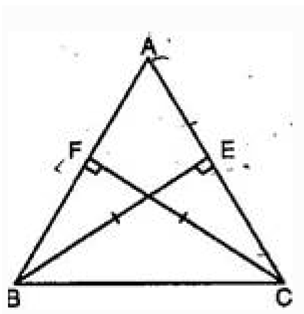


Answer.

In $\triangle ABE$ and $\triangle ACF$,
$\angle A = \angle A$ [Common]
$\angle AEB = \angle AFC = 90^{\circ}$ [Given]
$AB = AC$ [Given]
$\therefore \triangle ABE \cong \triangle ACF$ [By ASA congruency]
$\Rightarrow BE = CF$ [By C.P.C.T.]
$\Rightarrow$ Altitudes are equal.

Ex 7.2 Question 4.

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that:
(i) $\triangle ABE \cong \triangle ACF$
(ii) $AB = AC$ or $\triangle ABC$ is an isosceles triangle.

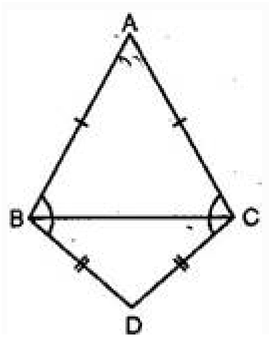


Answer.

(i) In $\triangle ABE$ and $\triangle ACF$,
$\angle A = \angle A$ [Common]
$\angle AEB = \angle AFC = 90^{\circ}$ [Given]
$BE = CF$ [Given]
$\therefore \triangle ABE \cong \triangle ACF$ [By ASA congruency]
(ii) Since $\triangle ABE \cong \triangle ACF$
$\Rightarrow BE = CF$ [By C.P.C.T.]
$\Rightarrow$ ABC is an isosceles triangle.

Ex 7.2 Question 5.

ABC and DBC are two isosceles triangles on the same base BC (See figure). Show that $\angle \mathbf{ABD} = \angle ACD$



Answer.

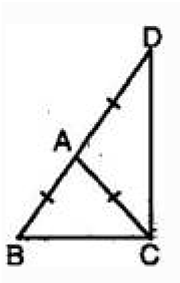
In isosceles triangle ABC,
$\mathrm{AB}=\mathrm{AC}$[Given]
$\angle \mathrm{ACB}=\angle \mathrm{ABC}$ (i) [Angles opposite to equal sides]

Also in Isosceles triangle BCD.
$BD=DC$
$\therefore \angle BCD=\angle CBD$ (ii) [Angles opposite to equal sides]

Adding eq. (i) and (ii),
$\angle \mathrm{ACB}+\angle \mathrm{BCD}=\angle \mathrm{ABC}+\angle \mathrm{CBD}$
$\Rightarrow \angle \mathrm{ACD}=\angle \mathrm{ABD}$
or $\angle \mathrm{ABD}=\angle \mathrm{ACD}$

Ex 7.2 Question 6.

$\triangle ABC$ is an isosceles triangle in which $AB=AC$. Side BA is produced to D such that $AD=AB$. Show that $\angle BCD$ is a right angle (See figure).



Answer.

In isosceles triangle ABC,
$AB=AC$[Given]
$\angle \mathrm{ACB}=\angle \mathrm{ABC}$.(i) [Angles opposite to equal sides]

Now $AD=AB$[By construction]

But $\mathrm{AB}=\mathrm{AC}$[Given]
$\therefore AD=AB=AC$
$\Rightarrow \mathrm{AD}=\mathrm{AC}$

Now in triangle ADC,
$AD=AC$
$\Rightarrow \angle \mathrm{ADC}=\angle \mathrm{ACD}$ (ii) [Angles opposite to equal sides]

Since $\angle \mathrm{BAC}+\angle \mathrm{CAD}=180^{\circ}$ (iii) [Linear pair]

And Exterior angle of a triangle is equal to the sum of its interior opposite angles.
$\therefore$ In $\triangle \mathrm{ABC}$,
$\angle \mathrm{CAD}=\angle \mathrm{ABC}+\angle \mathrm{ACB}=\angle \mathrm{ACB}+\angle \mathrm{ACB}$ [Using (i)]

$\Rightarrow \angle \mathrm{CAD}=2\angle \mathrm{ACB}$

Similarly, for $\triangle \mathrm{ADC}$,
$\angle \mathrm{BAC}=\angle \mathrm{ACD}+\angle \mathrm{ADC}$
$=\angle \mathrm{ACD}+\angle \mathrm{ACD}=2\angle \mathrm{ACD}$

From eq. (iii), (iv) and (v),
$2\angle \mathrm{ACB}+2\angle \mathrm{ACD}=180^{\circ}$
$\Rightarrow 2(\angle \mathrm{ACB}+\angle \mathrm{ACD})=180^{\circ}$
$\Rightarrow \angle \mathrm{ACB}+\angle \mathrm{ACD}=90^{\circ}$
$\Rightarrow \angle \mathrm{BCD}=90^{\circ}$

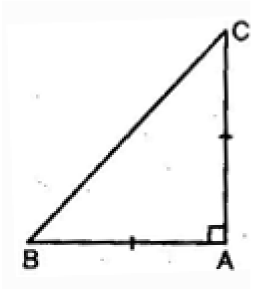
Hence $\angle \mathrm{BCD}$ is a right angle.

Ex 7.2 Question 7.

$\mathbf{ABC}$ is a right angled triangle in which $\angle \mathbf{A}=90^{\circ}$ and $\mathbf{AB}=\mathbf{AC}$. Find $\angle \mathbf{B}$ and $\angle \mathbf{C}$.

Answer.

ABC is a right triangle in which,



$\angle A = 90^\circ$ And $AB = AC$
In $\triangle ABC$
$AB = AC$
$\Rightarrow \angle C = \angle B \ldots\ldots\ldots.$ (i)

We know that, in $\triangle ABC$,
$\angle A + \angle B + \angle C = 180^\circ$ [Angle sum property]
$\Rightarrow 90^\circ + \angle B + \angle B = 180^\circ$
[$\angle A = 90^\circ$ (given) and $\angle B = \angle C$ (from eq. (i)]
$\Rightarrow 2\angle B = 90^\circ$
$\Rightarrow \angle B = 45^\circ$

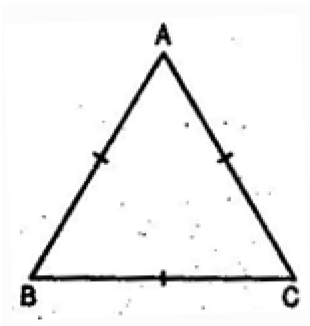
Also $\angle C = 45^\circ [\angle B = \angle C]$

Ex 7.2 Question 8.

Show that the angles of an equilateral triangle are 60° each.

Answer.

Let ABC be an equilateral triangle.



$\therefore AB = BC = AC$
$\Rightarrow AB = BC$
$\Rightarrow \angle C = \angle A \ldots$
Similarly, $AB = AC$
$\Rightarrow \angle C = \angle B$
From eq. (i) and (ii),
$\angle A = \angle B = \angle C$
Now in $\triangle ABC$
$\angle A + \angle B + \angle C = 180^\circ \ldots\ldots\ldots$ (iv)
$\Rightarrow \angle A + \angle A + \angle A = 180^\circ$
$\Rightarrow 3\angle A = 180^\circ$
$\Rightarrow \angle A = 60^\circ$
Since $\angle A = \angle B = \angle C$[From eq. (iii)]
$\therefore \angle A = \angle B = \angle C = 60^\circ$

Since $\angle A = \angle B = \angle C$ [From eq. (iii)]

Hence each angle of equilateral triangle is 60°.

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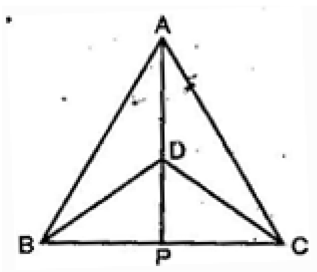
Ex 7.3 Question 1.

$\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show that:



(i) $\triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$
(ii) $\triangle \mathbf{ABP} \cong \triangle \mathrm{ACP}$
(iii) AP bisects $\angle \mathbf{A}$ as well as $\angle \mathbf{D}$.
(iv) AP is the perpendicular bisector of BC.

Answer.

(i) $\triangle \mathrm{ABC}$ is an isosceles triangle.
$\therefore AB = AC$
$\triangle \mathrm{DBC}$ is an isosceles triangle.
$\therefore BD = CD$

Now in $\triangle \mathrm{ABD}$ and $\triangle \mathrm{ACD}$,
$AB = AC$[Given]
$BD = CD$[Given]
$\mathrm{AD} = \mathrm{AD}$[Common]
$\therefore \Delta \mathrm{ABD} \cong \triangle \mathrm{ACD}$ [By SSS congruency]
$\Rightarrow \angle \mathrm{BAD} = \angle \mathrm{CAD}$ [By C.P.C.T.]

(ii) Now in $\triangle \mathrm{ABP}$ and $\triangle \mathrm{ACP}$,
$AB = AC$[Given]
$\angle \mathrm{BAD} = \angle \mathrm{CAD}$[From eq. (i)]
$\mathrm{AP} = \mathrm{AP}$
$\therefore \Delta \mathrm{ABP} \cong \triangle \mathrm{ACP}$ [By SAS congruency]
(iii) Since $\triangle \mathrm{ABP} \cong \triangle \mathrm{ACP}$ [From part (ii)]
$\Rightarrow \angle \mathrm{BAP} = \angle \mathrm{CAP}$[By C.P.C.T.]
$\Rightarrow$ AP bisects $\angle \mathrm{A}$.

Since $\triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$[From part (i)]
$\Rightarrow \angle \mathrm{ADB} = \angle \mathrm{ADC}$ [By C.P.C.T.]

Now $\angle \mathrm{ADB} + \angle \mathrm{BDP} = 180^{\circ}$ [Linear pair]

And $\angle \mathrm{ADC} + \angle \mathrm{CDP} = 180^{\circ}$ [Linear pair]

From eq. (iii) and (iv),
$\angle \mathrm{ADB}+\angle \mathrm{BDP}=\angle \mathrm{ADC}+\angle \mathrm{CDP}$

$\Rightarrow \angle \mathrm{ADB}+\angle \mathrm{BDP}=\angle \mathrm{ADB}+\angle \mathrm{CDP}[\text{ Using (ii)}]$
$\Rightarrow \angle \mathrm{BDP}=\angle \mathrm{CDP}$
$\Rightarrow$ DP bisects $\angle \mathrm{D}$ or AP bisects $\angle \mathrm{D}$.
(iv) Since $\triangle ABP \cong \triangle ACP$[From part (ii)]
$\therefore BP=PC[By\ \mathrm{C.P.C.T.}]$

And $\angle \mathrm{APB}=\angle \mathrm{APC}$[By C.P.C.T.] (vi)

Now $\angle \mathrm{APB}+\angle \mathrm{APC}=180^{\circ}$ [Linear pair]
$\Rightarrow \angle \mathrm{APB}+\angle \mathrm{APC}=180^{\circ}$ [Using eq. (vi)]
$\Rightarrow 2\angle \mathrm{APB}=180^{\circ}$
$\Rightarrow \angle \mathrm{APB}=90^{\circ}$
$\Rightarrow \mathrm{AP} \perp \mathrm{BC}$

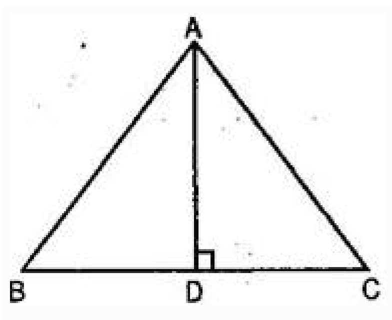
From eq. (v), we have BP PC and from (vii), we have proved AP $\perp$. So, collectively AP is perpendicular bisector of BC.

Ex 7.3 Question 2.

AD is an altitude of an isosceles triangle ABC in which $AB=AC$. Show that:
(i) AD bisects BC.
(ii) AD bisects $\angle \mathrm{A}$.

Answer.

In $\triangle ABD$ and $\triangle ACD$,
$AB=AC$[Given]
$\angle \mathrm{ADB}=\angle \mathrm{ADC}=90^{\circ}[\mathrm{AD} \perp \mathrm{BC}]$

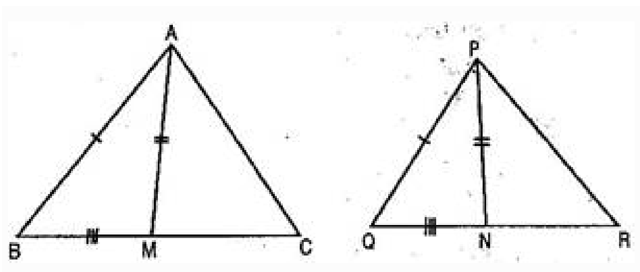


$\mathrm{AD}=\mathrm{AD}$ [Common]
$\therefore \triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}[\mathrm{RHS}$ rule of congruency]
$\Rightarrow BD=DC$ [By C.P.C.T.]
$\Rightarrow AD$ bisects BC

Also $\angle \mathrm{BAD}=\angle \mathrm{CAD}$ [By C.P.C.T.]
$\Rightarrow$ AD bisects $\angle \mathrm{A}$.

Ex 7.3 Question 3.

Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of $\triangle$ PQR (See figure). Show that:



(i) $\triangle$ ABM $\cong$ $\triangle$ PQN
(ii) $\triangle \mathrm{ABC} \cong \triangle \mathrm{PQR}$

Answer.

AM is the median of $\triangle \mathrm{ABC}$.
$\therefore BM=MC=\frac{1}{2}BC$
PN is the median of $\triangle \mathrm{PQR}$.

Now $\mathrm{BC}=\mathrm{QR}$ [Given] $\Rightarrow \frac{1}{2}\mathrm{BC}=\frac{1}{2}\mathrm{QR}$
$\therefore BM=QN$
(i) Now in $\triangle \mathrm{ABM}$ and $\triangle \mathrm{PQN}$,
$AB=PQ$[Given]
$\mathrm{AM}=\mathrm{PN}$[Given]
$\mathrm{BM}=\mathrm{QN}$[From eq. (iii)]
$\therefore \triangle \mathrm{ABM} \cong \triangle \mathrm{PQN}$ [By SSS congruency]

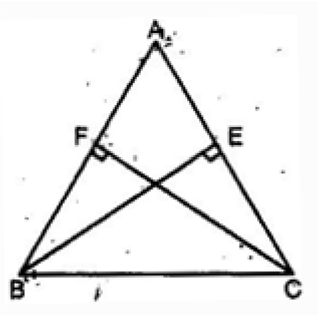
$\Rightarrow \angle B = \angle Q$ [By C.P.C.T.]
(ii) In $\triangle ABC$ and $\triangle PQR$,
$AB = PQ$ [Given]
$\angle B = \angle Q$ [Prove above]
$BC = QR$[Given]
$\therefore \triangle ABC \cong \triangle PQR$ [By SAS congruency]

Ex 7.3 Question 4.

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Answer.

In $\triangle BEC$ and $\triangle CFB$,



$\angle BEC = \angle CFB$ [Each 90°]
$BC = BC$ [Common]
$BE = CF$ [Given]
$\therefore \Delta BEC \cong \Delta CFB$ [RHS congruency]
$\Rightarrow EC = FB$ [By C.P.C.T.] ....(i)

Now In $\triangle AEB$ and $\triangle AFC$
$\angle AEB = \angle AFC$ [Each 90°]
$\angle A = \angle A$[Common]
$BE = CF$ [Given]

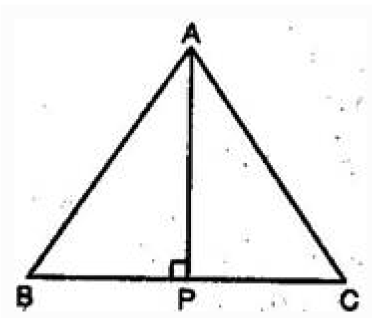
$\therefore \Delta AEB \cong \Delta AFC$ [ASA congruency]
$\Rightarrow AE = AF$[By C.P.C.T.]
Adding eq. (i) and (ii), we get,
$EC + AE = FB + AF$
$\Rightarrow AB = AC$
$\Rightarrow ABC$ is an isosceles triangle.

Ex 7.3 Question 5.

ABC is an isosceles triangles with $AB = AC$. Draw $AP \perp BC$ and show that $\angle B = \angle C$.

Answer.

Given: ABC is an isosceles triangle in which $AB = AC$



To prove: $\angle B = \angle C$

Construction: Draw AP $\perp BC$

Proof: In $\triangle ABP$ and $\triangle ACP$
$\angle APB = \angle APC = 90^\circ$ [By construction]
$AB = AC$ [Given]
$AP = AP$ [Common]
$\therefore \triangle ABP \cong \triangle ACP$ [RHS congruency]
$\Rightarrow \angle B = \angle C$ [By C.P.C.T.]